ME 597/747
Autonomous Mobile Robots

Mid Term Exam

Duration: 2 hour
Total Marks: 100

Instructions: Read the exam carefully before starting. Equations are at the back, but they are NOT necessarily valid for all problems. The exam is closed book.

Multiple-Choice – 2 marks awarded for each correct answer, 1 mark deducted for each incorrect answer, 0 marks if no answer.

1. Which of the following robots are FULLY autonomous robots:
   a) Ventana ROV (for Underwater jelly-tracking)
   b) Minerva
   c) MARs Rover Spirit
   d) None of the above

2. Behavior based navigation is better than planning based navigation in some situations because:
   a) It uses a world model
   b) Behaviors cannot be fused through competitive coordination schemes
   c) It navigates by planning a path around obstacles.
   d) None of the above

3. In general, a locomotion system can possibly have
   a) Good stability and Good controllability and Good maneuverability
   b) Good controllability and Good maneuverability
   c) Good stability and Good controllability
   d) None of the above

4. A GPS array needs:
   a) 2 satellites to resolve 3 unknowns
   b) 4 satellites to resolve 4 unknowns
   c) 3 satellites to resolve 3 unknowns
   d) 5 satellites to resolve 5 unknowns

5. Dead reckoning is a good way to navigate because
   a) It does not require proprioceptive sensors
   b) Odometry errors are non-deterministic
   c) Low-resolution encoders have noise
   d) None of the above
6. A robot using control inputs to predict position estimates is moving in the negative y direction in a global coordinate frame. With respect to the variance in its final position estimate, it will have:
   a) Greatest variance in the y direction
   b) Equal variance in the x and y directions
   c) Least variance in the direction orthogonal to its motion
   d) None of the above

7. In comparing single-query vs. multi-query Probabilistic Road Map (PRM) planning algorithms, which is true
   a) Single query is faster than multi-query
   b) The query step of multi-query planning is slow relative to single query planning
   c) In order to build a trajectory satisfying kinematic constraints, both require post-processing to smooth the trajectories.
   d) All of the above

8. In single – query planning, clustering occurs when
   a) Milestones are expanded by considering the dynamics of the vehicle.
   b) Milestones selected for expansion are randomly picked with probability inverse to the local roadmap milestone density
   c) Milestones selected for expansion are randomly picked with probability inverse to the number of milestones in the roadmap
   d) Milestones are expanded by randomly selecting control inputs to the robot

9. Which properties are true about graph searches with branching factor b and depth d
   a) Depth first searches always have O(bd) time complexity
   b) Depth first searches always take O(bd) space complexity
   c) Breadth first searches always take O(bd) time complexity
   d) Breadth first searches always take O(bd) space complexity

10. For A*, an admissible heuristic is one that
    a) Always over-estimates the path cost to the start node
    b) Always under-estimates the path cost to the start node
    c) Always under-estimates the path cost to the goal node
    d) None of the above
Short Answer – Total of 5 marks awarded for each question.

11. Laser range finders (e.g. SICK) are one of the most powerful range sensors on the market.
   a) Describe how laser range finding sensor works when phase shift measurements are used to obtain the distance to an object. Be sure to use a diagram.
   b) Explain how ambiguous results might occur and the method for resolving this ambiguity

12. The Mark III mini-sumo robot is equipped with inexpensive range sensors.
   a) Describe how this technology works. To aid this description, use a diagram and derive (from basic geometry) the equation used to obtain the distance.
   b) Draw the sensors characteristic curve that maps range to output voltage.
   c) Using the diagrams in a) and b), discuss limitations of this technology.

   a) Describe how a stochastic map represents the world.
   b) Consider a situation where a mobile robot moves along a straight path from location 1 to location 2, and then from location 2 to location 3. These motions can be described by the vectors \( y_{12} \) and \( y_{23} \) respectively. At location 1, the robot senses object 1 with measurement \( z_{1a} \). At location 2, the robot senses object 1 with measurement \( z_{1b} \), and a new object 2 with measurement \( z_{2a} \). At location 3, the object can only detect object 2 with measurement \( z_{2b} \). For each location, provide the stochastic map using compound relationship operators.

   Assume that measurements can be fused with the Kalman filter operator KF. For example the state \( h \) could be obtained from fusing two measurements \( m_1 \) and \( m_2 \):
   \[
   h = m_1 \text{ KF} m_2
   \]

   Step 1: \( x_r=0, x_1=z_{1a} \)
   Step 2: \( x_r=y+0, x_1=y_{12}+z_{1a} \text{ K } z_{1b}, x_2=y_{12}+z_{2a} \)
   Step 3: \( x_r=y+ya, x_1=y_{12}+z_{1a} \text{ K } z_{1b}, x_2=y_{23}+y_{12}+z_{2b} \text{ KF } y_{12}+z_{2a} \)

   c) Draw the corresponding confidence ellipses on the diagram below. Use a global coordinate frame attached to the workspace with (0, 0) being location 1. Assume the variance in range to the objects is greater than the variance in relative bearing to the object.
14. Single-Query Probabilistic Road Maps (PRMs) can be used to search configuration spaces with many degrees of freedom. We wish to set up a single-query PRM for a free-floating robot operating in a bounded, 3D environment. Assume the robot has 6 on/off thrusters (one at the center of each face of a symmetrical robot) with no rotation control. To set this up, address the following issues:

a) Define the degrees of freedom of the configuration space to search.
\[ C = \{x, y, z\} \]

b) Roadmap node selection: Describe how to select the nodes in the roadmap for expansion.
Discretize and pick from occupied cells, then pick a milestone from the cell.

c) Node generation: Given a node has been selected for expansion, describe a method of generating a new node for the roadmap. Include the dynamics of the robot, but don’t worry about collision-checking. Note the thrusters can only provide two thrusts: 0 or \( f_{\text{max}} \).
Pick \( u_x \) from \([-f_{\text{max}}, 0, f_{\text{max}}]\).

Short Math – Total of 5 marks awarded for each question.
15. A robot uses a laser range finder to obtain range and bearing estimates of the four legs of a square table. The data set is \( \{r_i, \alpha_i ; i = 1..4\} \). The confidence in this data can be characterized by the variance of range and bearing measurements \( (\sigma_r, \sigma_\alpha) \). Assume the robot was at position \((0, 0)\) in global coordinates when it took the measurements. Also assume the table is defined by \( X = [x \ y \ w] \), where the first two states define the position of the center of the table in global coordinates and the last state is the width of the table. (We neglect orientation to simplify the problem).

a) Define an equation of the table \( X = T(r_1, \alpha_1, r_2, \alpha_2, r_3, \alpha_3, r_4, \alpha_4) \)
\[ x = (r_1 \cos \alpha_1 + r_2 \cos \alpha_2 + r_3 \cos \alpha_3 + r_4 \cos \alpha_4)/4 \]
\[ y = (r_1 \sin \alpha_1 + r_2 \sin \alpha_2 + r_3 \sin \alpha_3 + r_4 \sin \alpha_4)/4 \]
\[ w = 0.25 \left[ (r_1 \cos \alpha_1 - r_2 \cos \alpha_2)^2 + (r_1 \sin \alpha_1 - r_2 \sin \alpha_2)^2 \right]^{1/2} + 0.25 \left[ (r_2 \cos \alpha_2 - r_3 \cos \alpha_3)^2 + (r_2 \sin \alpha_2 - r_3 \sin \alpha_3)^2 \right]^{1/2} \]
\[ 0.25 \left[ (r_3 \cos \alpha_3 - r_4 \cos \alpha_4)^2 + (r_3 \sin \alpha_3 - r_4 \sin \alpha_4)^2 \right]^{1/2} + \\
0.25 \left[ (r_4 \cos \alpha_4 - r_1 \cos \alpha_1)^2 + (r_4 \sin \alpha_4 - r_1 \sin \alpha_1)^2 \right]^{1/2} \]

b) What is the associated covariance matrix \( \Sigma_X \).

\[ C_X = \nabla f_{r, \alpha} C_{r, \alpha} \nabla f_{r, \alpha}^T \]

\[ C_{r, \alpha} = \text{diagonal}(\sigma_r, \sigma_\alpha, \sigma_r, \sigma_\alpha, \sigma_r, \sigma_\alpha) \]

\[ f_{r, \alpha} = \begin{bmatrix} dx/dr_1 & dx/d\alpha_1 & dx/dr_2 & dx/d\alpha_2 & dx/dr_3 & dx/d\alpha_3 \ldots \\
& dy/dr_1 & dy/d\alpha_1 & dy/dr_2 & dy/d\alpha_2 & dy/dr_3 & dy/d\alpha_3 \ldots \\
& & dw/dr_1 & dw/d\alpha_1 & dw/dr_2 & dw/d\alpha_2 & dw/dr_3 & dw/d\alpha_3 \ldots \end{bmatrix} \]

16. To measure the forward velocity of a two wheeled motorcycle robot, encoders with resolution \( g \) counts per revolution are used along with a cheap micro-processor to measure time (variance \( \sigma_t \)). Assume the robot always moves in a straight line, the wheel radius is \( r \) and the encoders on the front and back wheels each have their own variance \( \sigma_{e\text{-front}} \) and \( \sigma_{e\text{-back}} \) respectively.

a) How could you measure the velocity of each wheel \( i \). Provide an equation for both \( v_i \) and corresponding variance \( \sigma_i \). At each measurement, the robot will measure \( n \) encoder counts and \( t \) seconds.

\[ v_i = \frac{n}{g} \times 2 \pi r / t \]

\[ \sigma_i = \nabla f_{r,t} C \nabla f_{r,t}^T \]

\[ C = \text{diagonal}(\sigma_{e\text{-front}}, \sigma_{e\text{-back}}) \]

\[ \nabla f_{r,a} = \begin{bmatrix} dv/dn & dx/dt \end{bmatrix} \]

b) Combine the two wheel velocity measurements to get the velocity and its variance of the robot.

\[ v = v_{\text{front}} + K v_{\text{front}} \]

Kalman covariance

17. In most Markov localization, a key step implements Baye’s Rule.

a) Derive Baye’s rule using the product rule from statistical probability theory.

b) Write the equation used for the “Observation Prediction” step in Markov Localization that is based on Baye’s rule and explain how each term is calculated.
18. A robot stops to take range measurements to a wall. It gains several measurements at different bearings. Assume the wall is to be modeled by a line defined by \((r, \alpha)\), (see diagram below.)

a) Set up a weighted least squares problem to solve for the line equation parameters \((r, \alpha)\) that will minimize the perpendicular distance from each measurement point to the line.

b) Solve for \(r\).

![Diagram showing a robot taking range measurements to a wall.](image)

**Long Math – Total of 10 marks awarded for each question.**

19. Consider a differential-drive robot similar to the Mark III.

a) Given the robots initial state \(p = [x \ y \ \theta]\), use dead reckoning to estimate the resultant state \(p'\) after the two robot wheels are controlled to move distances \(\Delta s_{\text{left}}\) and \(\Delta s_{\text{right}}\) respectively.

b) Derive the corresponding covariance \(\Sigma_p\). Explain where terms come from.

20. Stereo vision is a powerful tool because the geometry allows for feature position measurements. Assuming an idealized setup (i.e. parallel cameras).

a) Draw a typical camera setup, be sure to include the coordinate system origin. Label all relevant distances.

b) Derive stereo vision geometry equations. That is, provide the feature position \([x \ y \ z]\) as a function of the camera image measurements \([x_r \ y_r]\) and \([x_l \ y_l]\) for the right and left cameras respectively.

**Algorithm Iterations – Total of 5 marks awarded for each question.**

21. Given below is a 3x3 BW intensity image. Perform the first three steps of the Zlog algorithm. What feature is apparent?

<table>
<thead>
<tr>
<th>0.1</th>
<th>0.1</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>0.1</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9/16</th>
<th>19/16</th>
<th>23/16</th>
</tr>
</thead>
<tbody>
<tr>
<td>19/16</td>
<td>51/16</td>
<td>61/16</td>
</tr>
<tr>
<td>23/16</td>
<td>61/16</td>
<td>55/16</td>
</tr>
</tbody>
</table>
22. The A* algorithm is being used to plan a resolution optimal path through a building hallway system. Given the map below, show the first 5 iterations of the A* algorithm by numbering the cells in the order that they are explored. To get you started, the first iteration will has been done. In your exam booklet, provide details of the iterations (including the first). Details should include the fringe set and their path cost breakdowns \( f(n), g(n), h(n) \). Finally, show the final path.

1. Fringe Set = \{ (2,8) \ f = 1+rt(85) \} \quad \text{Pick (2,8)}
2. Fringe Set = \{ (2,7) \ f = 2+rt(72), (3,8) \ f = 2+rt(74) \} \quad \text{Pick (2,7)}
3. Fringe Set = \{ (3,8) \ f = 2+rt(74), (2,6) \ f = 3+rt(61) \} \quad \text{Pick (3,8)}
4. Fringe Set = \{ (2,6) \ f = 3+rt(61), (4,8) \ f = 3+rt(65) \} \quad \text{Pick (2,6)}
5. Fringe Set = \{ (4,8) \ f = 3+rt(65), (1,6) \ f = 4+rt(74) \} \quad \text{Pick (4,8)}

Some Equations that might be useful:

\[ d = c \frac{t}{2} \]
\[ \lambda = \frac{c}{f} \]
\[ D = \frac{f l}{x} \]
\[ \Delta f = 2 ft v \cos \theta / c \]
\[ 1/f = 1/d + 1/e \]

\[ R = ld/(2e) \]

\[ E(x, y, t) = E(x + u \delta t, y + v \delta t, t + \delta t) \]

\[ p(A \land B) = p(A \mid B) \cdot p(B) \]

\[ E[X_1 X_2] = E[X_1] \cdot E[X_2] \]

\[ \text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) \]

\[ \rho_i \cos (\theta_i - \alpha) - r = d_i \]

\[ Y_j = f_j(X_1 \ldots X_n) \]

\[ C_Y = \nabla f_x \cdot C_x \cdot \nabla f_x^T \]

\[
P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\]

\[
G = \begin{bmatrix} 0.06 & 0.13 & 0.06 \\ 0.13 & 0.24 & 0.13 \\ 0.06 & 0.13 & 0.06 \end{bmatrix}
\]

\[ f(n) = g(n) + h(n) \]

\[ q = q_1 + K (q_2 - q_1) \]

\[ K = \sigma_i^2 / (\sigma_i^2 + \sigma_x^2) \]

\[ \sigma^2 = \sigma_i^2 / (\sigma_x^2 + \sigma_x^2) \]

\[ \Delta \theta = (\Delta s_{\text{right}} - \Delta s_{\text{left}}) / b \]

\[ \begin{bmatrix} \sin(\alpha + \beta) & -\cos(\alpha + \beta) & -\cos(\beta) \end{bmatrix} R(\theta) \begin{pmatrix} \xi \end{pmatrix} = r \varphi \]

\[ \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) & \sin(\beta) \end{bmatrix} R(\theta) \begin{pmatrix} \xi \end{pmatrix} = 0 \]

\[ x_{ij} \oplus x_{jk} = \begin{bmatrix} x_{jk} \cos \theta_{ij} - y_{jk} \sin \theta_{ij} + x_{ij} \\ x_{jk} \sin \theta_{ij} + y_{jk} \cos \theta_{ij} + y_{ij} \end{bmatrix} \begin{bmatrix} \theta_{ij} + \theta_{jk} \end{bmatrix} \]
\[ x_{ij} = \begin{bmatrix} -x_{ij} \cos \theta_{ij} - y_{ij} \sin \theta_{ij} \\ x_{ij} \sin \theta_{ij} - y_{ij} \cos \theta_{ij} \\ -\theta_{ij} \end{bmatrix} \]

\[
p(x_t | o_t) = \sum_{x'} p(x_t | x'_{t-1}, o_t) p(x'_{t-1})
\]

\[
p(x_t | z_t) = \frac{p(z_t | x_t) p(x_t)}{p(z_t)}
\]